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The Strong, Electromagnetic and Weak Couplings [and Discussion]

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The strong, electromagnetic and weak couplings

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A brief review, aimed at non-specialists, is given of present knowledge of the parameters needed to describe strong and electroweak interactions at energies up to a few hundred GeV. Empirical evidence that the strong and electroweak couplings converge to a single 'grand unified' coupling at much higher energies is reviewed and the uncertainties in the calculation of the proton lifetime in the minimal SU(5) theory are discussed.

1. Strong interactions

There is general agreement that strong interactions are described by quantum chromodynamics, which is the only sensible theory which fits the facts (for an introduction to QCD and references see Llewellyn Smith 1982). QCD describes the interaction of coloured quarks through the exchange of coloured spin 1 'gluons', as indicated more or less directly by the data, especially the evidence for chiral symmetry. There are also gluon self interactions, which are required for the theory to make mathematical sense.

The lagrangian of QCD appears to contain several parameters: the strong coupling constant (g_s), quark masses (m_q) and a mysterious parameter θ. I shall ignore θ, which is known to be less than 1.5×10^{-9}; presumably there is some principle, outside QCD, which forces it to be very small or zero. In considering those hadrons whose flavour quantum numbers are carried by u and d quarks (i.e. with strangeness, charm, etc., equal to zero), it is likely that we can ignore the existence of heavier quarks to a good approximation. It is also an excellent approximation to set $m_{u,d} = 0$. In this case the Lagrangian exhibits exact chiral symmetry which is realized in the Nambu–Goldstone mode, leading to $m_\pi = 0$ (a measure of symmetry breaking is $m_\pi^2/m_\rho^2 = 0.03$) and many other predictions which work to better than 10 %.

With just u and d and $m_{u,d} = 0$, the only remaining parameter is g_s. However, g_s depends on the energy, E, at which it is measured. Consequently we can replace the dimensionless parameter g_s by a parameter E_0 with units of energy or inverse length (in the usual $\hbar = c = 1$ units), for example, we can define E_0 by $g_s(E_0) \equiv 1$. However, as there are no other dimensional quantities in the theory, E_0 is not actually a parameter, but simply the unit of energy. Everything else is calculable in principle in terms of E_0, e.g. m_p/E_0, m_ρ/E_0 are calculable and so, therefore, is $g_s(m_p)$! Thus, with only u and d quarks QCD has *no* parameters in the chiral limit!

The idea that couplings such as g_s are energy dependent is so important for QCD and for attempts at grand unification, discussed below and in the accompanying talks by Weinberg and Ellis, that I shall indicate briefly how it comes about. Consider a classical test charge placed in a dielectric. Its charge $Q(R)$, defined in terms of the electric flux out of a sphere surrounding it, depends on the radius, R, of the sphere. For R much smaller than the typical intermolecular distance it will have the same value, Q, as in free space, but at larger distances there is screening, due to a net flow of negative polarization charge into the sphere, and $Q(R)$ decreases to Q/ϵ. Likewise in field theory vacuum polarization makes charges distance (or equivalently energy) dependent. In QED, for example, the contribution of virtual particles (through e^+e^- loops, etc.)

to the photon propagator causes the effective value of α, which controls the electromagnetic force between two charged particles, to increase from its value at infinity of $(137.036\ldots)^{-1}$ as the particles approach one another. For small four-momentum transfer, q^2, the dominant terms give

$$\alpha_{\text{eff}}(q^2) = \alpha - (\alpha^2/15\pi)\,(q^2/m_e^2),$$

which contributes a very well verified -27 MHz to the Lamb shift.

A similar phenomenon occurs in all field theories, except that in the case of non-Abelian theories such as QCD the coupling *increases* with distance or, equivalently, decreases with increasing energy (Gross & Wilczek 1973; Politzer 1973). For example, in QCD with four flavours of quarks

$$g^2(E) = 48\pi^2/(25\ln(E^2/\Lambda^2)),$$

to leading order at large E. Here we see explicitly that the value of the dimensionless quantity g is determined by a dimensional parameter Λ, which sets the scale at which strong interactions are strong (a precise definition of Λ requires that we go beyond leading order and depends on technical details of how the theory is formulated; below we quote values of $\Lambda = \Lambda_{\overline{\text{MS}}}$ for the so-called barred minimal subtraction ($\overline{\text{MS}}$) scheme).

The idea of calculating g_s, or equivalently calculating Λ/M where M is a directly observable quantity with dimensions of mass which can be used as a scale, has actually been realized in lattice QCD. In this formulation, fields are only defined at discrete points on a lattice in space–time or on the links between them (for reviews and references see Creutz *et al.* 1983; Kogut 1982; Rebbi 1982), the intention being to let the lattice spacing, a, tend to zero at the end of the calculation. For finite a, the theory can be simulated on a computer. So far, only the gluons have been treated as dynamical degrees of freedom in most calculations. Heavy quarks can be introduced as static sources and the calculations indicate that the force between them corresponds to a quark-confining potential $V = \sigma R$ at large R even for $a \rightarrow 0$. For given a, the coupling $g(a)$ is adjusted to give $\sigma \approx (0.43\,\text{GeV})^2$, as required by the spectroscopy of heavy quark systems, or $\sigma \approx (0.50\,\text{GeV})^2$, which gives the observed slope of Regge trajectories in the string model. The quantity that corresponds to $g_s(E)$ in the continuum can be calculated in terms of $g(a)$ and it is found (Creutz *et al.* 1983) that

$$\Lambda_{\overline{\text{MS}}} = (0.23 \pm 0.05)\,\sigma^{\frac{1}{2}} \approx 110\,\text{MeV},$$

thus giving $g_s(E)$ *absolutely* ($\sigma^{\frac{1}{2}}$ serving as the unit of energy). There has also been some encouraging progress in calculating hadron masses on the lattice (see Creutz *et al.* 1983; Rebbi 1983) but the overall size of the lattices used so far has been too small (typically 1 fm† or less compared with the root mean square radius of the proton which is 0.8 fm) for the results to be expected to be realistic. Furthermore the calculations do not include 'dynamical' fermions in virtual loops (although there are some arguments which suggest that their effects will be small).

The value of $\alpha_s(E)$ can be measured in some high energy experiments for which predictions can be made with QCD perturbation theory (for a recent review with references to the original papers see Altarelli 1982). Large E is a necessary condition for perturbation theory to be used but in general it is not sufficient. Mathematically, the perturbation expansion is spoiled by terms such as $(\alpha_s(E)\ln(E/m))^n$ in most cases; the divergence as $m \rightarrow 0$ shows physically that the result is sensitive to long distance physics so that it depends on how hadrons are constructed from quarks and gluons, which is obviously not described by perturbation theory. However, there are some processes for which factors of $\ln E/m$ do not occur that are insensitive to how hadrons are made of

† 1 fm $= 10^{-15}$ m.

quarks and gluons (e.g. σ ($\bar{e}e \rightarrow$ hadrons) and the Q^2 dependence of deep inelastic structure functions) which can presumably be treated perturbatively. It is hard to extract α_s from the data for these processes as it is not very small, so perturbation theory converges slowly, and it is even harder to extract Λ because α_s is only sensitive to Λ at small E, where it is large and there are important incalculable subasymptotic contributions, of relative size μ^2/E^2. Various experiments, whose results all agree qualitatively at least with the predictions of perturbative QCD, give $\Lambda_{\overline{\mathrm{MS}}}$ in the range 100–350 MeV (for recent results see Eisle 1982), not in disagreement with the lattice calculations.

To conclude on the strong interactions: QCD is also certainly correct. In the light quark sector, it has no parameters in the limit $m_{u,d} = 0$. The quark masses (for light and heavy quarks) must be introduced as parameters from outside QCD: the big question being, what determines m_q/Λ? The main challenge for theorists is to show that QCD really leads to the known hadrons with the properties observed.

2. Electroweak interactions

We can no longer discuss the electromagnetic and weak interactions separately. Indeed, the differential cross section for $e^+e^- \rightarrow \mu^+\mu^-$, long thought of as a testing ground for QED, disagrees with pure QED at high energies, although the departure from pure QED is well described by interference with the expected neutral current contribution (for a review see Davier 1982).

The parameters in the standard $SU(2) \times U(1)$ electroweak gauge theory fall into two categories (for a recent review of electroweak gauge theories see Beg & Sirlin 1982):

(i) the gauge couplings g_1 and g_2 or, equivalently, α_{em} and $\sin^2\theta_W$;

(ii) parameters associated with symmetry breaking, i.e. couplings of Higgs bosons in the canonical model: the Higgs self couplings, which determine m_H and $\langle\phi\rangle$ (which in turn determines m_W and m_Z), and the Yukawa couplings which determine the quark masses and Cabbibo–Kobayashi–Maskawa mixing angles. I shall assume the simplest possible symmetry breaking scheme in which $m_W = m_Z \cos\theta_W$ to lowest order.

To lowest order, processes which do not involve the Higgs boson directly can be described by three parameters, in addition to fermion masses and mixings, which can be chosen to be α, $\sin^2\theta_W$ and the Fermi constant G_F. A very large amount of data is well described in terms of these parameters. In particular, different experiments give consistent values of $\sin^2\theta_W$; the values determined by the most accurate experiments are shown in the accompanying table. As stressed particularly by Veltman (Veltman 1980; Green & Veltman 1980) a vital question is whether this agreement survives electroweak radiative corrections; tests which are sensitive to second and higher order effects are the electroweak equivalent of the Lamb shift, corrections to the muon's g factor and the other classic tests of QED (for reviews see Wheater 1982; Aoki *et al.* 1982). Nominally, second order effects shift $\sin^2\theta_W$ by an amount of order ± 0.02; in fact detailed calculations give shifts which are somewhat less for the experiments shown in the table and do not destroy the agreement between different measurements (beyond the lowest order the value of the parameter $\sin^2\theta$ depends on how it is defined; the values in the table are for the $\overline{\mathrm{MS}}$ scheme with scale m_W). Another way to express the magnitude of second order effects is to compare the values $m_W = 78.2^{+2.7}_{-2.5}$ and $m_Z = 89.0^{+2.2}_{-2.0}$ derived from $\nu N \rightarrow \nu X$ in lowest order and $m_W = 83.1^{+3.1}_{-2.8}$, $m_Z = 93.8^{+2.5}_{-2.2}$ derived taking electroweak corrections to the experiment and to the mass formula into account (Wheater & Llewellyn Smith 1982). Clearly the experiments are becoming sensitive

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to second order effects† and incisive tests of SU(2) × U(1) will be possible in the next few years with $\nu e \to \nu e$, $\nu N \to \nu X$, $ep \to eX$, $e^+e^- \to \mu^+\mu^-$, m_W, m_Z among other quantities.

These measurements probe the idea that the W and Z are fundamental gauge bosons. If they are composite the same predictions can be obtained to leading order in a large class of theories (Bjorken 1979) but the properties of the W and Z would differ to second order, unless the binding energy is very large. These measurements are also sensitive to the contributions of new relatively light particles, such as the plethora of new particles with masses of order m_W whose existence is predicted in supersymmetric theories, which would alter the radiative corrections at the per cent level (for details see Schwarzer 1983).

Table 1. Values of $\sin^2\theta_W$

(Determined (*a*) with the Born approximation for the electroweak interactions (column 1) and (*b*) by including one loop corrections in the $\overline{\text{MS}}$ scheme with scale m_W (column 2); the corrected values are taken from Wheater & Llewellyn Smith (1982) for νN and ed (νN has also been treated by Marciano & Sirlin (1981) with the same result). The corrections for m_W are based on Marciano & Sirlin (1980). The Born value for νN is based on an average of experiments. The value for ed is from the fit of Kim *et al.* (1981) to the experiment of Prescott *et al.* (1979). The value of m_W is from Arnison (1983*a*).)

	$\sin^2\theta_W$	$\sin^2\bar{\theta}_W(m_W)$
$\nu N \to \nu X$	0.227 ± 0.015	0.215 ± 0.015
$e_L^- d - e_R^- d$	0.223 ± 0.015	0.215 ± 0.015
$m_W = 81 \pm 5$	0.22 ± 0.03	$0.226^{+0.030}_{-0.026}$

To conclude on SU(2) × U(1): it is certainly correct to first approximation and even more incisive tests will soon be possible. However the conventional symmetry breaking mechanism (the Higgs effect) seems very *ad hoc* and is prolific in arbitrary parameters (m_W, m_Z, m_H, m_{q_i}, θ_{KM}). The challenging question is what fixes these parameters and why are the masses so disparate ($m_W/m_e \approx 1.6 \times 10^5$; $m_e/m_t < 2.5 \times 10^{-3}$, etc.)?

3. Grand unification?

The idea that $SU(3)_c$ (the gauge group of QCD), SU(2) and U(1) are subgroups of a single 'grand unifying' group is very appealing (for a review and references see Langacker 1981). If indeed strong and electroweak interactions are fundamentally the same, the definition of a hadron or lepton has no fundamental significance and it is natural to attempt to unify quarks and leptons by putting them in the same representation of the gauge group. If we assume that the fifteen states that form the 'first generation' of fermions (e_L^-, e_L^+, ν_L, $u_L^{r,b,g}$, $\bar{u}_L^{r,b,g}$, $d_L^{r,b,g}$, $\bar{d}_L^{r,b,g}$) form one, possibly reducible, representation it would follow that

$$g_2 = g_3,$$

if the symmetry were unbroken, where $g_{2,3}$ are the SU(2) and SU(3) couplings, and

$$g_1 \equiv (\tfrac{5}{3})^{\frac{1}{2}} g' = g_2 \quad \text{or} \quad \sin^2\theta_W = \tfrac{3}{8},$$

where g' is the conventionally defined U(1) coupling (Georgi *et al.* 1974). The 'grand symmetry' G must be broken by the gauge bosons corresponding to those generators of G which do not belong to SU(3) × SU(2) × U(1) acquiring very large masses (which we shall call generically m_X).

† Since this talk was given, the discovery of the Z was announced with (based on 5 events) $m_Z = 95.2 \pm 2.5$ GeV and an improved value of $m_W = 81 \pm 2$ GeV was given, based on 27 events (Arnison 1983*b*); the experimenters stress that final calibration of the calorimeter is still in progress and small scale shifts in these masses, most likely affecting both, are still possible.

These vector bosons couple quarks to leptons and mediate nucleon decay, to which we return later. For energies which are asymptotically large compared with m_X, the symmetry becomes exact. Thus the couplings behave as shown in figure 1. Note that g_1/g_2 decreases with energy so that

$$\sin^2\theta(E) = 3g_1^2(E)/(3g_1^2(E)+5g_2^2(E))$$

will be less than the symmetry value of $\frac{3}{8}$ at low energy.

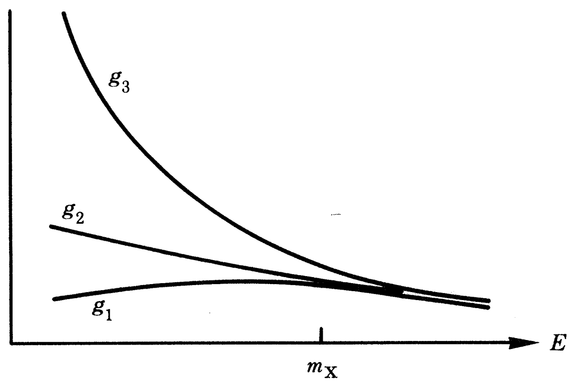

FIGURE 1

The precise way that the g_i approach each other for $E \gtrsim m_X$ depends on G. However, as a first approximation we can set $m_X = 0$ for $E > m_X$ and $m_X = \infty$ for $E < m_X$. This approximation, in which the g_i meet at $E = m_X$ and their evolution up to this point can be studied without knowledge of G (Georgi *et al.* 1974), gives $\sin^2\theta \approx 0.21$ at low energy, in good agreement with experiment, and m_X of order 10^{15} GeV.

To find more than the order of magnitude of m_X, and to obtain $\sin^2\theta_W$ precisely, it is necessary to specify the group G. The simplest choice is SU(5) (Georgi & Glashow 1974). The minimal version of this theory, with no ingredients for which there is no phenomenological necessity, gives

$$\sin^2\theta_{\overline{MS}}(m_W) = 0.215 \pm 0.006,$$

for $\Lambda_{MS} = 150^{-100}_{+250}$ MeV, in excellent agreement with the experimental value given above, and $m_X = (1.3^{+0.9}_{-0.6}) \times 10^{15}\Lambda_{\overline{MS}}$ (these quantities have been calculated independently by many authors; here we quote the results of Llewellyn Smith *et al.* 1981). Uncertainties in low energy data, used in evaluating the evolution of α_{em}, contribute ± 0.18 to the error in the coefficient 1.3; the error from 'three loop' contributions, which have not been calculated, is taken to be ± 0.15, which is probably reasonable as they are proportional to $\alpha_s(m_W)/\pi$; a further error of ± 30 % is introduced by allowing the very massive coloured Higgs bosons to range from $10^{-2}m_X$ to $10^{+2}m_X$. Given a value for m_X, we can calculate the nucleon decay rate using a model to calculate the matrix elements of the appropriate four fermion operator. We consider the decay $p \rightarrow \pi^0 e^+$, for which the best experimental limits exist, which is thought to be the dominant mode in SU(5). Of all credible model calculations, the quark model gives the longest lifetime but even so it is possible that it yields an underestimate. Somewhat arbitrarily introducing an error to cover an underestimate by a factor of two gives:

$$\Gamma(p \rightarrow \pi^0 e^+)^{-1} = (3^{+3}_{-?}) \left(\frac{m_X}{1.3 \times 0.15 \times 10^{15}\,\text{GeV}}\right)^4 \left(\frac{\ln(m_N/\Lambda)}{\ln(m_N/0.15)}\right)^{\frac{12}{21}} \times 10^{29}\ \text{years}$$

$$= (6^{+90}_{-?}) \left(\frac{\Lambda_{\overline{MS}}}{350\,\text{MeV}}\right)^4 \times 10^{30}\ \text{years},$$

where the first line is normalized to $\Lambda_{\overline{\text{MS}}} = 150$ MeV, for some time the standard theoretical guess, and the second to 350 MeV because there is some evidence that $\Lambda_{\overline{\text{MS}}} > 150$ MeV and 350 GeV is about the largest value currently thought to be acceptable (the median value is based on the quark model calculation of Isgur & Wise (1982)). We see that if $\Lambda_{\overline{\text{MS}}} \approx 350$ MeV, the current results (Goldhaber 1983) do not quite rule out SU(5). However, a limit greater than 10^{32} years could only be accommodated by a conspiracy of the uncertainties, and an even larger value of $\Lambda_{\overline{\text{MS}}}$, and would essentially eliminate the minimal version of SU(5).

Longer lifetimes can easily be obtained by changing the model, for example to SO(10), or to a supersymmetric GUT. However, if this is done the precise prediction for $\sin^2 \theta$ is lost; the predicted value is still of order 0.21 but it could be bigger or smaller by as much as ± 0.05, depending on new unknown parameters, although the measured value can still be accommodated in most models.

To conclude on grand unification: the underlying idea is very appealing and it is supported by the fact that the couplings do seem to merge at energies of order 10^{15} GeV. The minimal SU(5) model is strikingly successful in explaining the value of $\sin^2 \theta_W$ but it predicts a rate for $p \rightarrow \pi^0 e^+$ which seems to be on the verge of being ruled out. The lifetime can be made longer by altering the theory but this opens a Pandora's box of more complex alternatives.

4. General conclusions

The evidence for $SU(3) \times SU(2) \times U(1)$ is excellent: QCD is correct and the electroweak interactions are unified. However, this successful model is surely not the final theory: it has far too many arbitrary parameters, it does not explain the different 'generations' and it does not incorporate gravity. In any case the big problem for any more complete theory is the origin of masses, or equivalently the origin of symmetry breaking, and the origin of enormous ratios of masses such as m_W/M_P, where M_P is the Planck mass of 10^{19} GeV which characterizes gravity, or m_W/m_X.

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Discussion

H. B. Nielsen (*Bohr Institute, Copenhagen, Denmark*). How strong evidence for unification is it that the couplings of the gauge theories agree so well? Could one not obtain a similar prediction by having all three coupling constants coming for instance from a scheme like the one Steven Weinberg told us about? The prediction that the couplings are equal even might be dreamt to be not too difficult if one has any way of predicting them at all.

C. H. Llewellyn Smith. The fact that, suitably normalized, the couplings seem to converge at high energy provides evidence for a connection between the different interactions. It is of course possible that this connection has nothing to do with conventional grand unified theories.

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